Archimedes. Energy Spectra Analysis.
Fractals, multifractals, pseudo-fractals and noise.

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Introduction

Modern, high resolution, flown magnetic surveys began in the early 1990s. A number of technological advances brought about a significant advance in capability, these included:

- Differential GPS for improved location and navigation
- Advanced sensors improved sensitivity and resolution
- Base stations provided real time correction for diurnal variation and ionospheric noise
- Higher density of flight lines.
- Newer detectors provide vector of field, further improving resolution.

Taken together these improvements yielded at least a tenfold increase in information. The Archimedes processes take advantage of this, and are designed to extract the maximum value from this data.

Energy spectral analysis (ESA)

Understanding of the ESA process is rooted in the manner in which is simple single body disturbs the Earth's magnetic field.

In 1970 Spector and Grant provided the first analysis of the form of the spectral energy from a logically realistic distribution of magnetic sources. This analysis assumes a random distribution of individual blocks, with a random distribution of size. Their analysis showed, rather usefully, that such a distribution yielded a spectrum that has the same form as a single block at the same depth. It is thus possible to extend the techniques for a single block to an entire geo. horizon.

Analysis based upon Spector and Grant has been used for decades in determining the depth of large scale geological features, in particular, the depth of crystalline basement.

The range of sources that can be detected is considerable. In addition to crystalline basement, magnetic sources can appear in at leas the following scenarios:

- Volcanic flows
- Sedimentary layers where the source rock is magnetic
- Faults, where mineralisation occurs in the fault plane
- Fractures, similar to faults, where mineralization in the crack may be magnetic.
- Chemical processes. Especially along edges of salt formations, laminar bodies of magnetic material occur due the action of salt and water.

In addition to the estimation of depth, a thin laminar body will exhibit a spectrum that allows an estimation of its depth to be made.

The utility and accuracy of this method has been proven over a considerable number of projects. Figure 1, below, shows a typical spectrum, and line fit to the decay. This particular example was later ground truthed with a well, where the predicted depth of the horizon matched the well to within a metre.

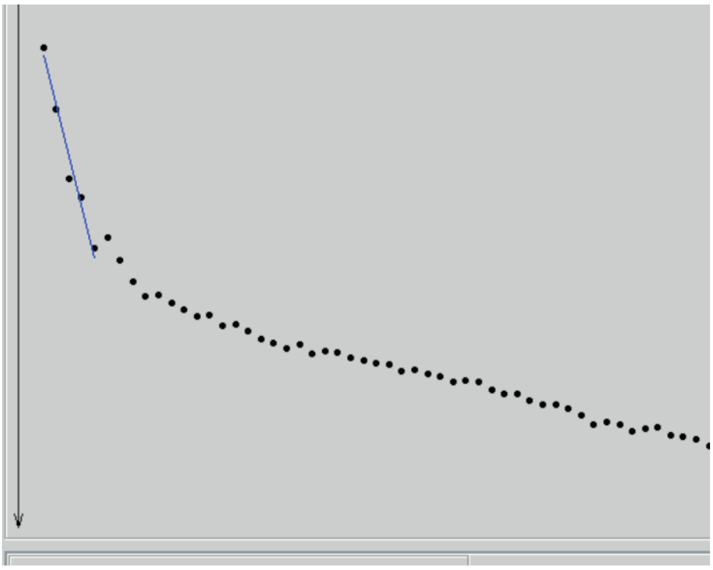

Figure 1. Energy Spectra Decay

Fractals

In 1982 Mandelbrot outlined ideas for the fractal nature of many natural processes, and later work has found evidence of many fractal geological processes.

Fractal processes are distinguished by their self-similar nature. That is that at different scales the same apparent process and appearance is present. Clearly, in real processes, as opposed to abstract ones, there are limits to the range of a fractal process, both at the largest and smallest scales.

For example turbulent flow is well known to be characterised by fractals, however the self-similarity of the flow is bounded at small scales by the increasing Reynolds number. Thus turbulent cells are not comprised of ever smaller turbulent cells, but at a critical scale the self-similarity halts. This has a bearing on the distribution of susceptibilities. There is good evidence that at large scales the distribution of susceptibilities is fractal in nature. However over smaller scales, typically those in interest in an individual survey, clear distinctions between otherwise homogeneous horizons are seen.

Charaterising the form of a fractal is done using a parameter that measures the amount of self similarity at different scales. Estimating this parameter is done with a number of different algorithms, and the success of the estimation depends in part upon the ability of the algorithm to match the form of the self similarity. The fractal parameters are termed the Hurst parameters.

A critical signature of a fractal process is that some measure of the fractal at any scale, when plotted against size displays a power curve. The power curve is itself self-similar (cut any part of the power curve out, and it can be rescaled to exactly match any other part of the power curve.) The parameters of the power curve are directly related to the Hurst parameters, and can be used interchangeably in many measures.

When performing spectral analysis, the shape of the energy spectrum from a fractal process will also follow a power curve, and the self similarity measure is similarly derivable.

However all of the above analysis assumes that a single dominating fractal process exists. When more than one process, ones with different scaling and self-similaries overlap, the system is termed multi-fractal. When this is the case the derivation of coefficients is much less clear cut.

A uniform random process fits within the range of fractal processes, it is trivially self similar at all scales, but is a degenerate form of fractal. It is charterised by a power curve that has a B of zero – that is a straight line.

Multi-Fractals

Fractal processes in geology are clearly important. However attempting to charaterize a particular geological setting with a single fractal process is unlikely to be realistic.

An important example can be seen when considering a volcanic flow. The most trivial form of a volcanic flow is one that flows to a uniform depth over a flat landscape, and is then buried under later sedimentary layers. Such a flow is not detectable as it has no magnetic signature. However, in reality the flow is not over a flat landscape, but over a landscape that has been subject to various erosive processes – which are well characterized as fractal processes. However, whilst one area may exhibit a typical hilly profile, it may abut a river valley, which itself displays fractal forms in the branching of tributaries. The two having different Hurst parameters. Later the top of the flow will itself be weathered, but due to the different nature of the rock, will exhibit different fractal forms to the landscape onto which the flow occurred. Later the horizon will be buried. The varying thickness of the flow will now exhibit a clear magnetic signature. However, it may have been subject to a number of different fractal processes, each with different signatures. Later, faulting or folding in the layers may impose further changes to its morphology. Thus, whilst most of the processes that lead to the resolvable magnetic signature have some level of fractal nature (folding for instance does not) the overall signature is the superposition of all of them.

The above is mostly seen in the 2D extent of a horizon. Depth is also subject to fractal processes. The time between volcanic flows, the time between climatic changes, rates of erosion, all display some level of fractal behavior. These are independent of the processes forming the 2D morphology of an horizon, but do affect its final depth. These processes are also independent, so the final result is again the superposition. Importantly for potential field analysis, the number of horizons that contain magnetic sources is much less than the total number of horizons. So whilst the pattern of horizon depths is coupled to fractal processes, the coupling to magnetic horizons is much less strong.

Scale and sampling limitations

There are two limits on the scale that is detectable in ESA. The flight line spacing of the survey provides a lower limit on the feature size that can be resolved, whilst the window size with which each spectra is calculated provides an upper limit. The Archimedes process uses the smallest possible window needed to detect a given horizon. This allows for the best possible spatial resolution as the window is moved across the horizon. Larger window sizes simply act to average the depth estimate over a wider area.

Features with a scale below that of the flight line spacing cannot be individually detected, however, clearly an ensemble of small-scale features can, and will, result in a larger scale magnetic signature that is detected. The important point is that the flight line spacing acts as a spatial filter over the area.

Work by Sergei Katsev, and Ivan L'Heureux (Computers & Geosciences 29 (2003) 1085–1089) has cast considerable doubt on claims of detection of fractal processes when the sample size is restricted. Indeed they show that for samples of less than 500 elements that the derivation of Hurst parameters is not statistically reasonable. Furthermore, they show that the presence of spikes or edges in even larger sample spaces leads to spurious Hurst values, which can lead to claims of fractal processes where none in fact exist. Clearly great care is needed in understanding the information in energy spectra.

Experience by Archimedes with ESA indicates that within the confines of a small window, one only just large enough to meaningfully detect energy from a profile, there is insufficient spatial sampling for a fractal distribution to dominate. Rather, the nature of the small sample is better approximated by a random one. As the extent of the horizon is imaged by moving the window, the horizon estimate may indeed display a fractal nature.

Other work

Work by Pilkington and Todoeschuck in 1999 is widely cited as proving that fractal processes dominate in ESA analysis. The work is however flawed for a number of reasons. They provide spectra for two areas of Canada. One is an area where the crystalline basement outcrops, and thus is at a known (zero) depth. The survey was gridded at 813m and flown at a height of 300m. Sadly it is not reasonable to attempt to detect a horizon with energy spectral analysis if the distance to the horizon is less than the line spacing. They present a spectra which incorrectly estimates the depth, which is not surprising. Such a failure is however not evidence for the dominance of fractal processes in the surveyed area. The work is difficult to verify, since neither the precise data set used is cited, not the processing steps used. However analysis of the provided spectra, and some comments made later in the paper about limitations in the process are disturbing. The spectra show a clear signature of the use of a poor quality radial averaging algorithm, and it would appear that the FFT used is a simple power of 2 algorithm with simplistic edge extension. The lack of care in edge extension and no mention of window function leads one to question the quality of the result. This is especially important for the upward continuation algorithms used in their work. Again, experience at Archimedes has continued to underline the crucial nature of the care needed with these choices, and the flaws present in many of the commonly used codes.

A uniformly fractal distribution of magnetic susceptibility would give a log energy spectrum with no straight line segments. Such line segments are often observed; hence typically the distribution cannot be uniformly fractal at all scales.

Signal to noise

An important pitfall in spectral analysis comes when the signal to noise ratio is poor, and in particular in areas where the signal has low absolute strength. Usually energy spectra are calculated from normalized energy, which is convenient for further processing and analysis. However, if there is very low energy the resultant spectra becomes dominated by the shape of the convolution of the window function used. Typically a Hann or half-Hann window is used in most processing. This yields a good balance between alias lobes in the final spectra and accuracy of frequency estimation. The convolution of this window yields to a spectra that may mimic a power law decay, and may lead to erroneous interpretation of the nature of the geomorphology, when there may simply be no, or very little magnetic signature.

More careful selection of window function can yield useful improvements in spectral resolution in difficult areas. Use of higher order window functions, such as 7^{th} order Kaiser Bessel are used in such circumstances.

Conclusions

Fractal processes are clearly important in geophysics. Many of these processes will influence the energy spectrum and result in a signature that may be recovered with spectral analysis. However, it is far from clear that spectra will be dominated by the influence of any one particular fractal distribution. As geology typically exhibits many distinct, overlapping fractal processes, when performing spectral analysis with windows of minimum size the original assumptions of random distributions in magnetic sources made by Spector and Grant is often the most reasonable. Experience with real surveyed data, corroborated by well log data, supports this view.