Fractals, Multi-Fractals, Psuedo-Fractals and Non-Fractals in Energy Spectral Techniques

Francis Vaughan

(Archimedes Consulting)

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Outline

- Fractals
- Fractal Processes
- Energy Spectrum Analysis
- Fractal and Non-Fractal Assumptions
- Window Size
- Noise
- MWT process
- Validation
- Conclusions

Acknowlegements

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  - Sam Yates
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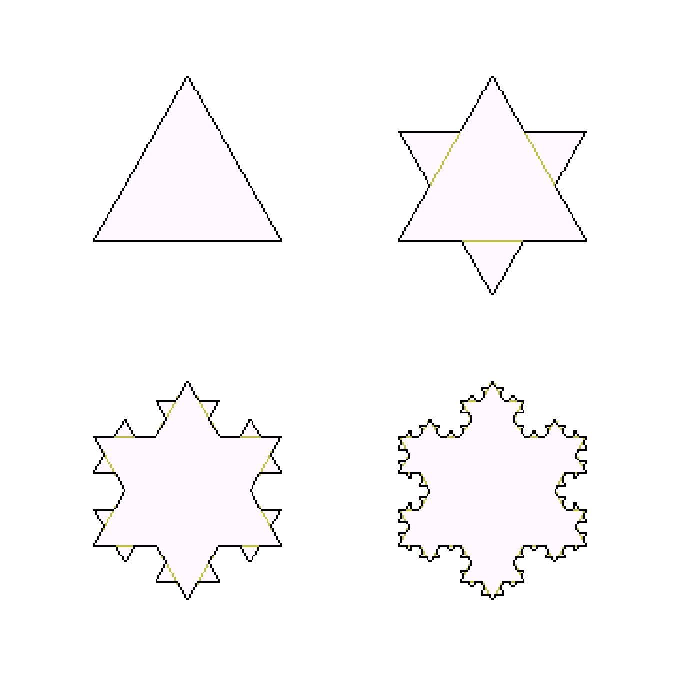
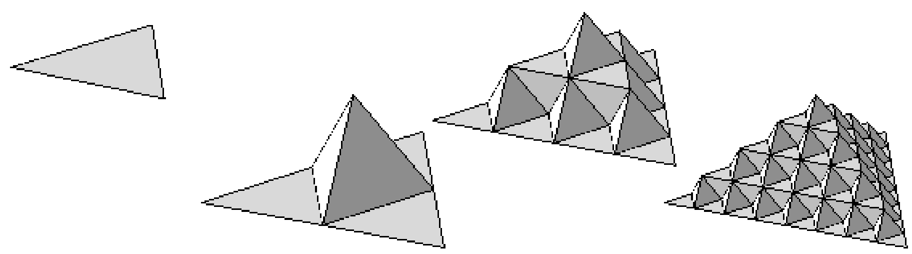
- **Thanks to:**
  - Scott Barnden
  - Craig Patten

Geometric Fractals

- Self similar geometry
- Repeated generation algorithm
- Non Integer Dimension

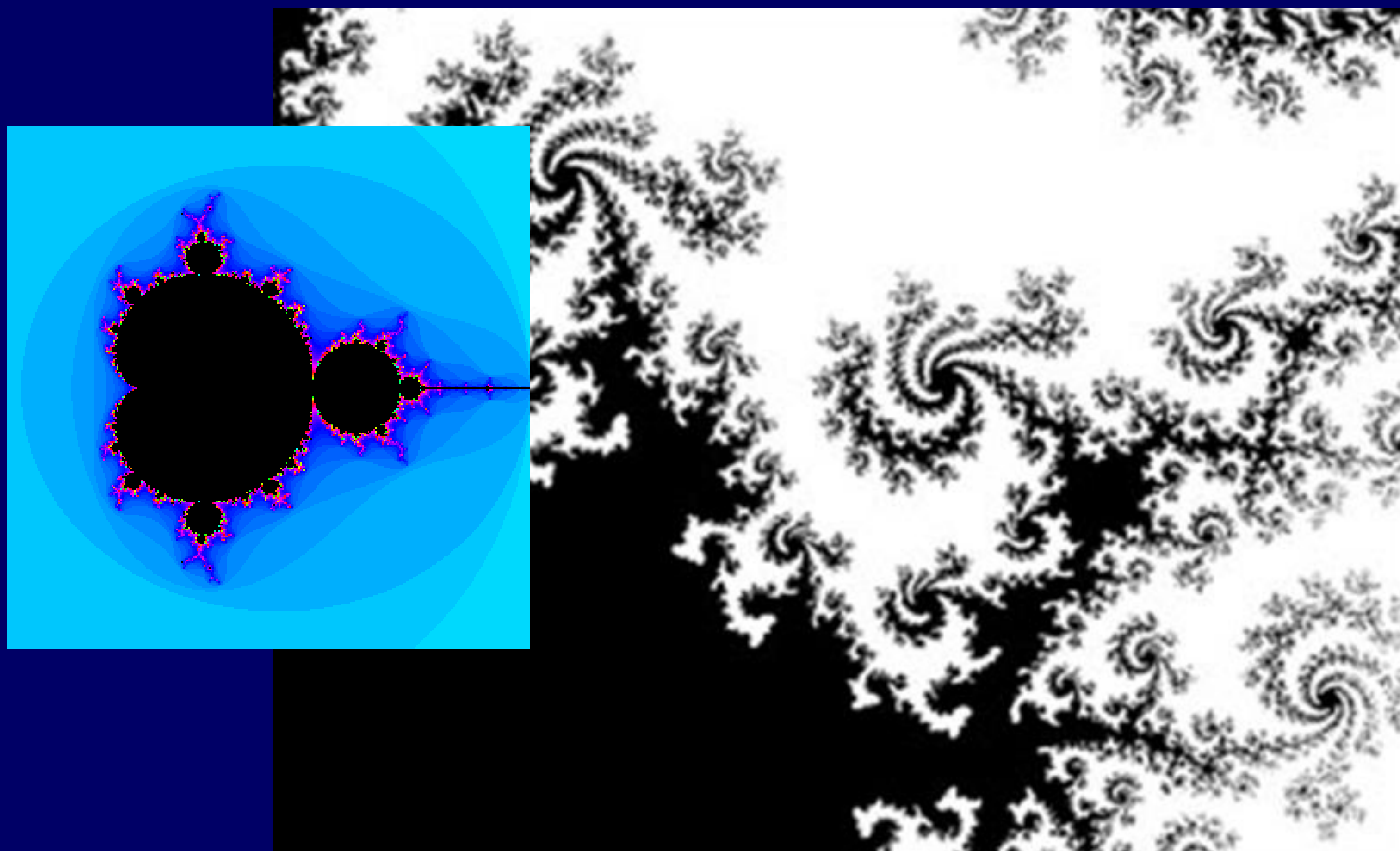


Koch Snowflake. 1904
D = 1.26

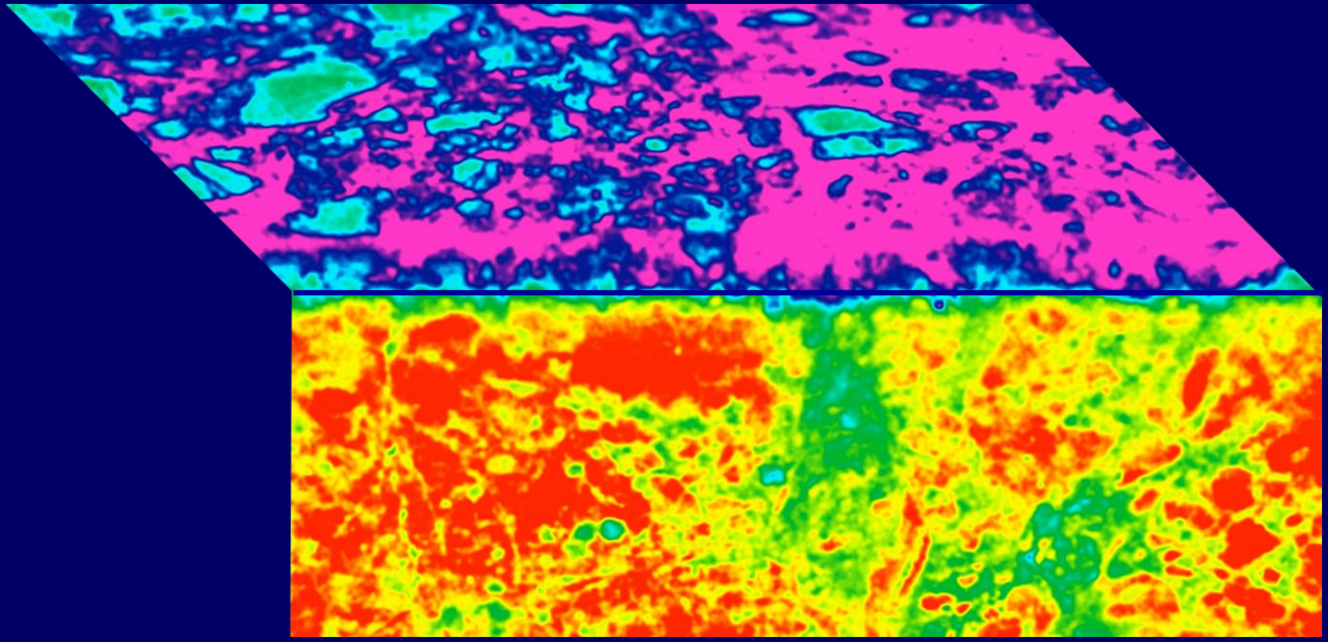


Fractals

- **The Fractal Geometry of Nature**
  Benoit B. Mandelbrot

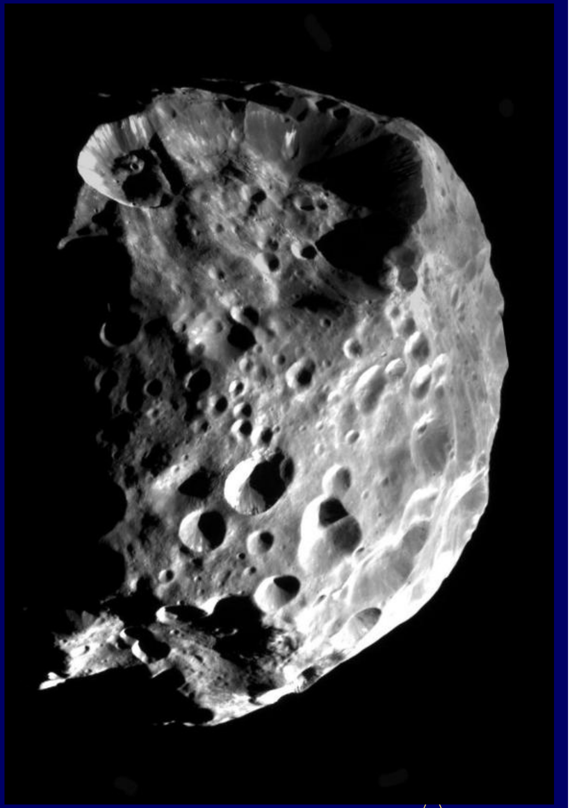


Scale Invariance



Scale Invariance 2

Craters



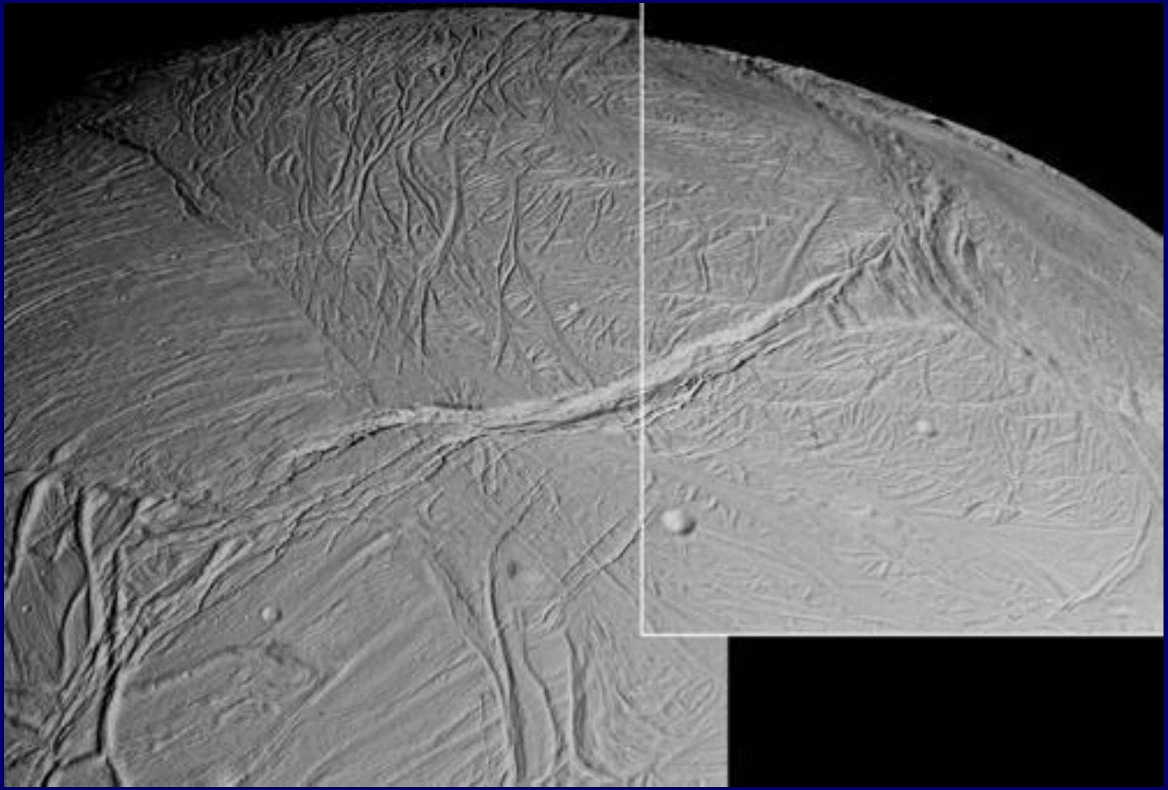
Craters - Fractal dimension

- The number N(>d) of impact craters having a diameter larger than d

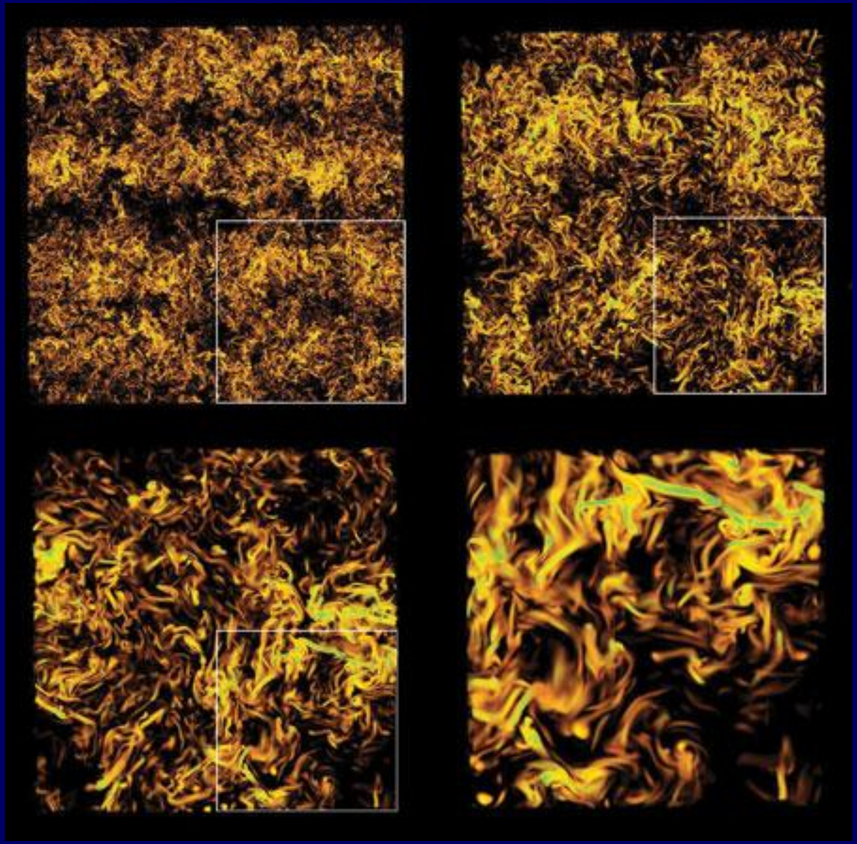
$$N(>d) \sim d^{-D}$$

- D is the fractal dimension
  - very close to 2.0 for the Moon, Mars and Venus.
  - The size of asteroids dimension D around 2.1.

Crustal Dynamics



Turbulence



Turbulence has fractal properties

Important for volcanic and magma flows

Susceptibility contrasts may follow

Image: © University Corporation for Atmospheric Research (UCAR)

Scale Limits

- Physical processes only work within given limits
- Turbulence
  - Reynolds number
- Tectonics
  - Plastic flow limits
  - Plate thickness

Anisotropy

- Scaling different in each dimension
- Different process in different dimensions
  - Topography
  - Sedimentary
- Non Fractal
  - Periodic forces (e.g. Milanchovitch Cycle)
  - Salt Tectonics
    - Non turbulent
    - Very low Reynolds Number

Anisotropy 2

- **Non-Linked**
- **Time varying**
  - **Large volume simple flow versus**
  - **Compound flows**

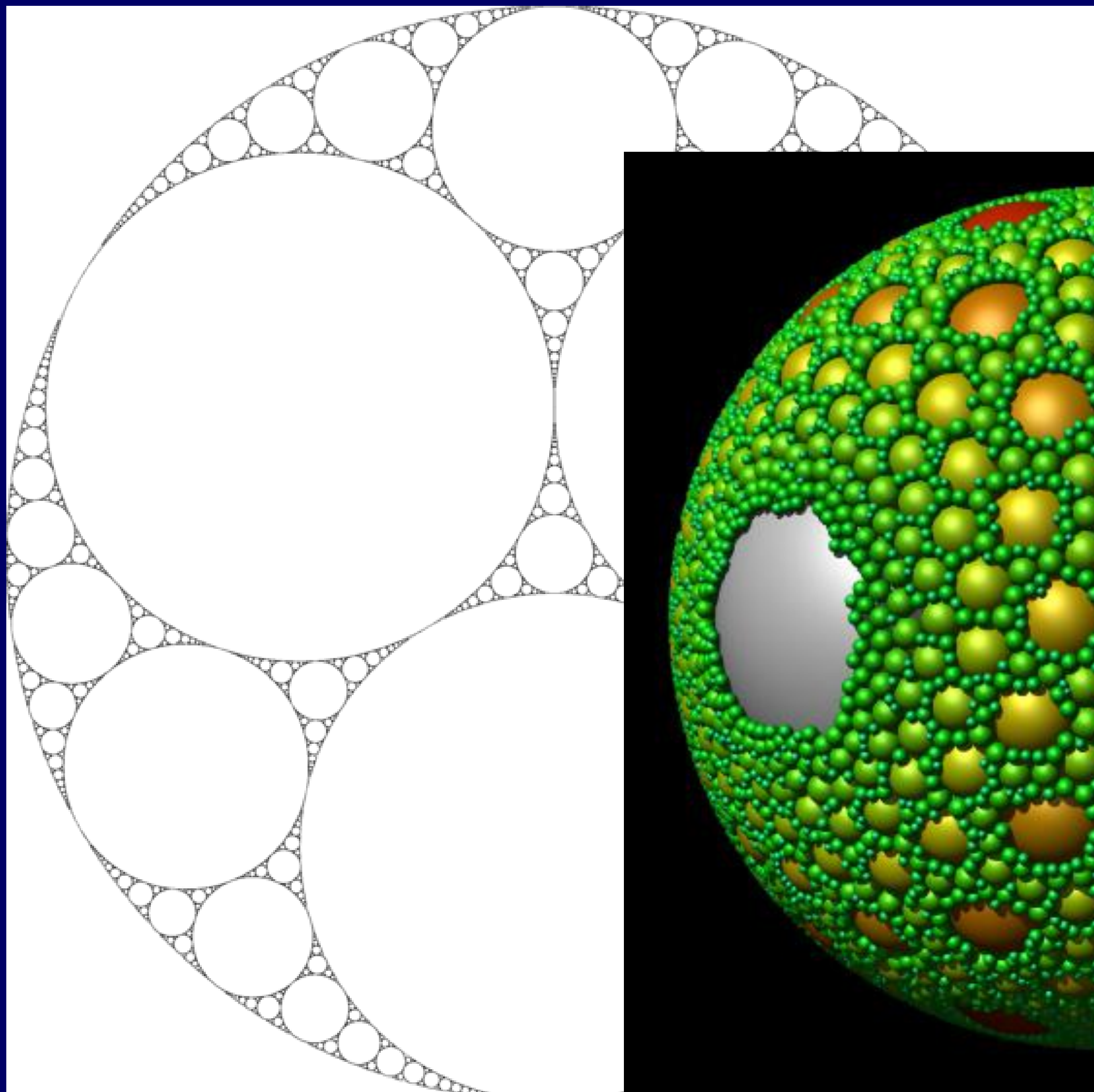
Scale

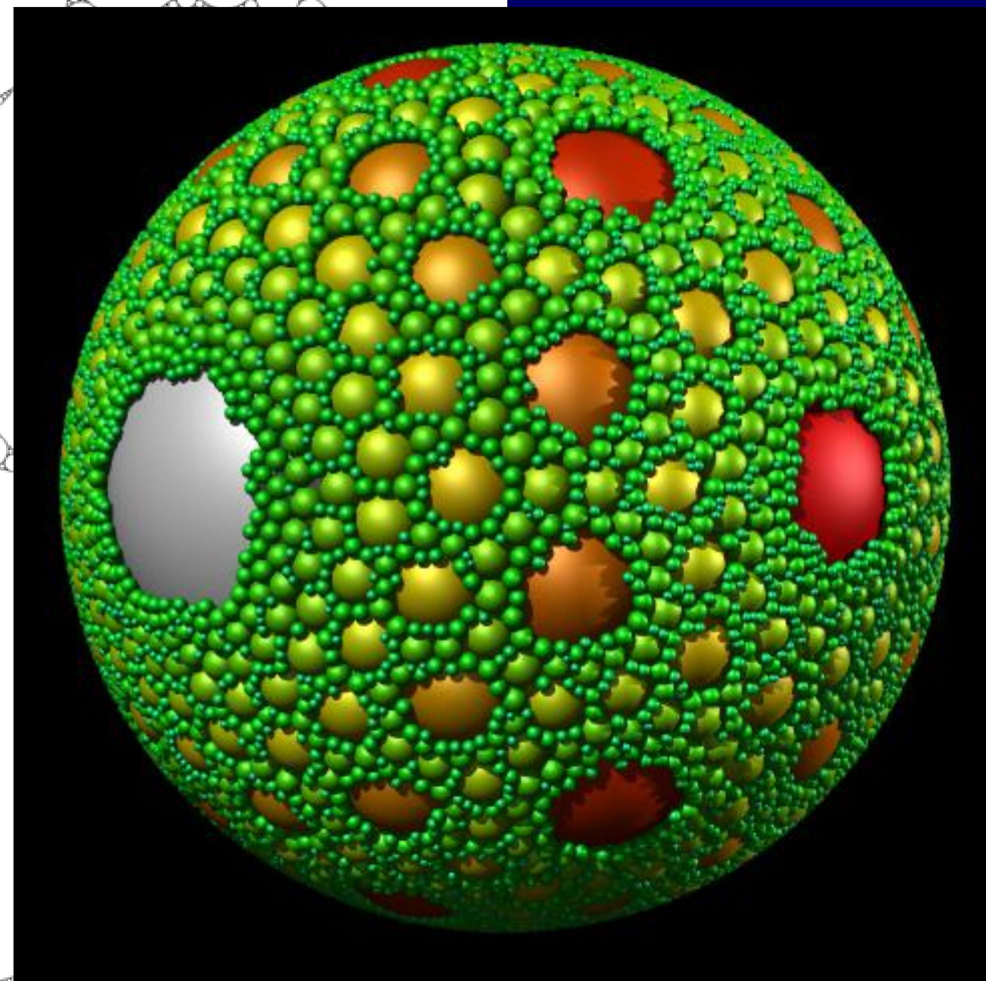
- Real processes only occur across a limited range of scales
- May be fractal within part of that range
- Other (possibly fractal) processes occur at other scales
- Processes may overlap in scale

Sampling

- Potential fields surveys provide limits to scales
- Impossible to see process with scale smaller than flight line spacing on gridded data
- Sub-sampling adds minimal information
- Gridding algorithms contribute in complex ways

Limits





Non and Psuedo Fractals

- Fractal $\longrightarrow$ Power law decay
- Power law $\not\longrightarrow$ Fractal

- "a power-law decay is not sufficient to identify a fractal distribution." Hough S.E. 1989.
  - Piecewise set of Gaussian processes yields power law

Non and Psuedo 2

- **Katsev and L'Heureux 2003**
  - Samples less than 500 elements not statistically valid for extracting fractal parameters
  - Spikes or discontinuities can cause false fractal dimension from fractal detection.
- **Fitting a line to log/log data is not assumption free**
  - Implicit model and fixing of invisible parameters

Models

- Desire for models that capture difficult phenomena
- Potential Fields
  - Simple Block Model
  - Simple Statistical Model
    - No to few parameters
  - Fractal - one (maybe) powerful parameter (more if anisotroptic)

- $M(p_1,p_2)\ \alpha\ R(p_1,p_2,p_3,\ldots.p_{big})$

  - $D_f = F(p_1,p_2,p_3,\ldots.p_{big})$ Fractal

Mathematical Models

- Desire for models that have tractable mathematics
  - Spector and Grant - linear - single parameter
  - Fractal - power law - single parameter
  - Euler deconvolution - assumption of single source per window
- All:
  - Simple
  - Wrong
  - Useful

Spector and Grant Model

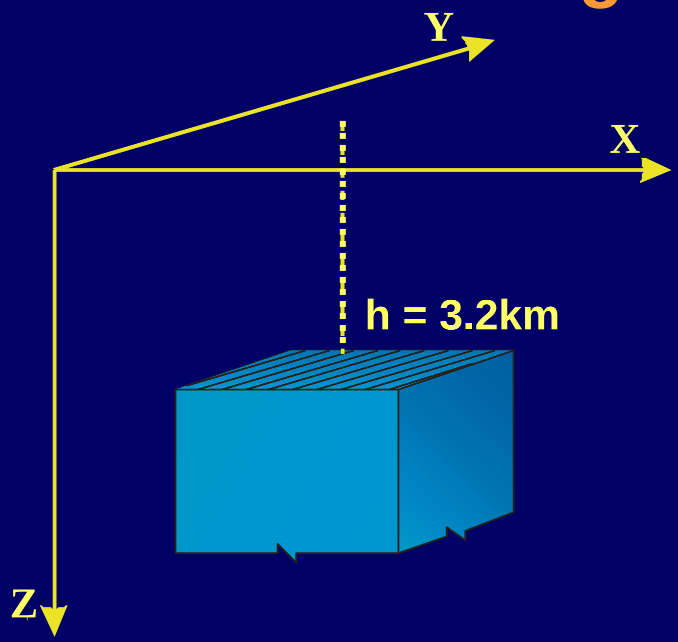
- **Magnetic interface is modeled by a statistical layer of magnetized vertical blocks.**
- **Horizon has correlation of blocks**

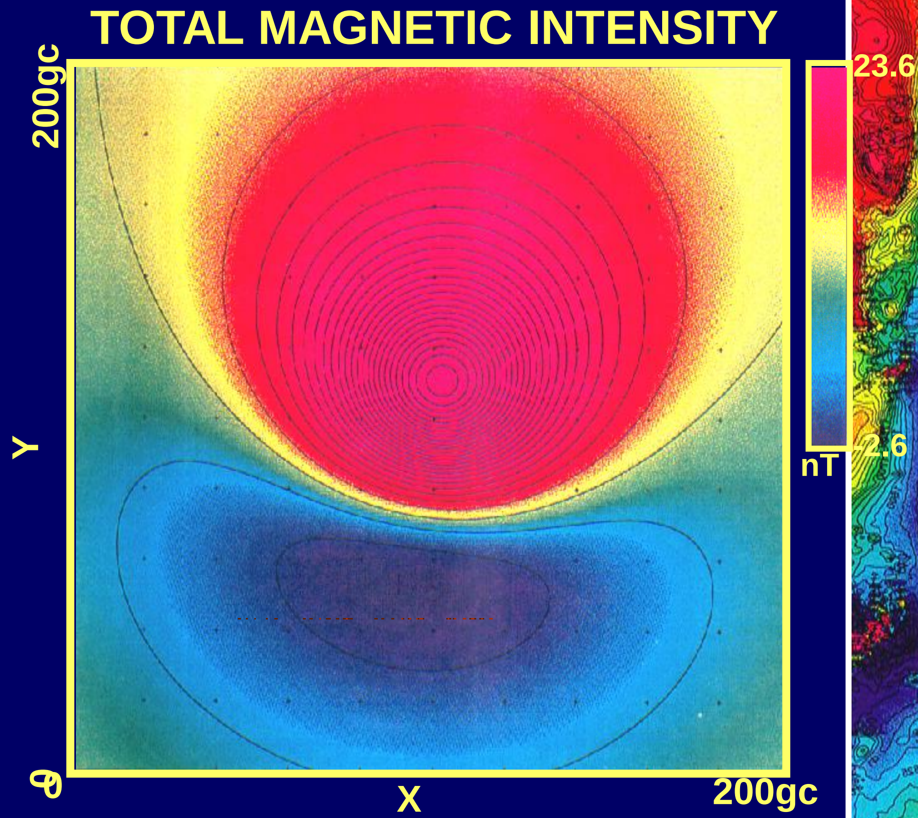
$$E(\rho) \square e^{-2h\square\rho} (1- e^{-t\square\rho})^2 \square S(\rho)$$

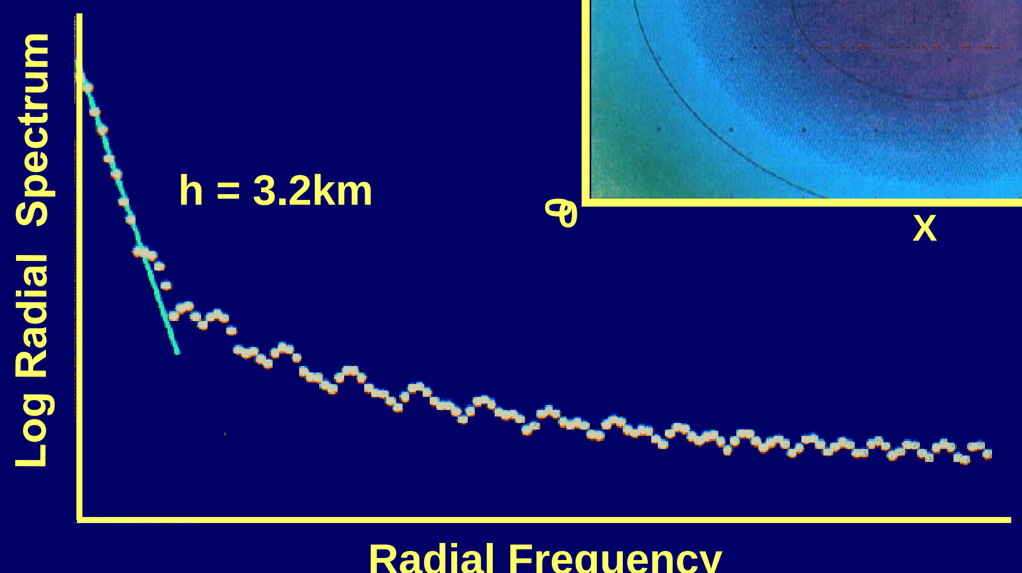
h = depth to top

t = thickness

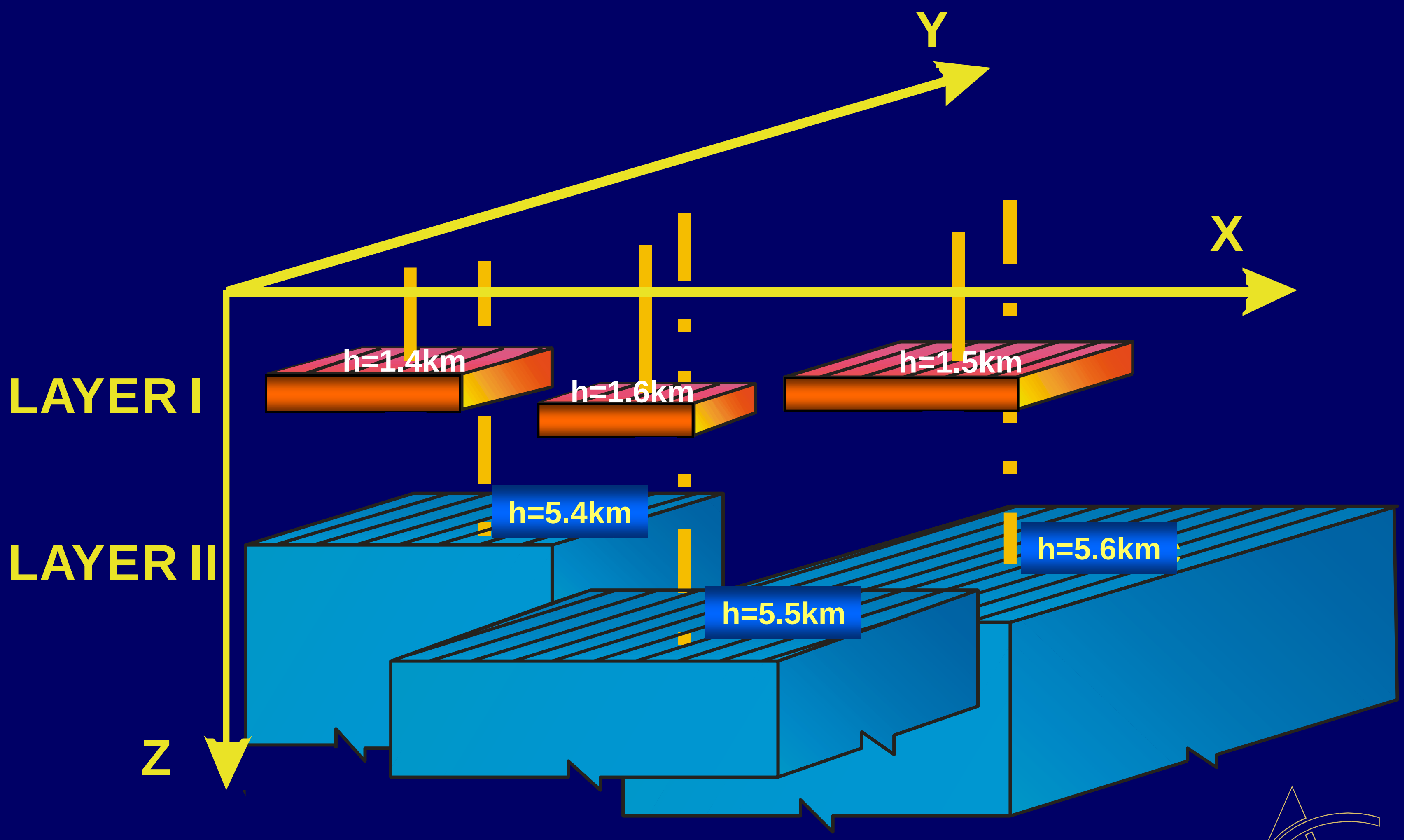
Single Prism Model



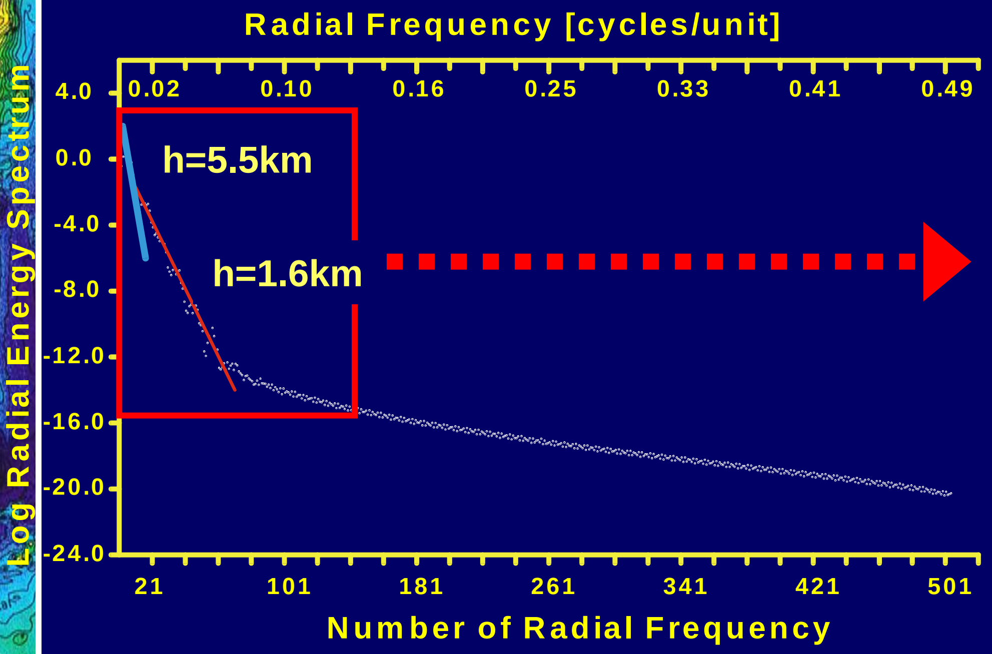


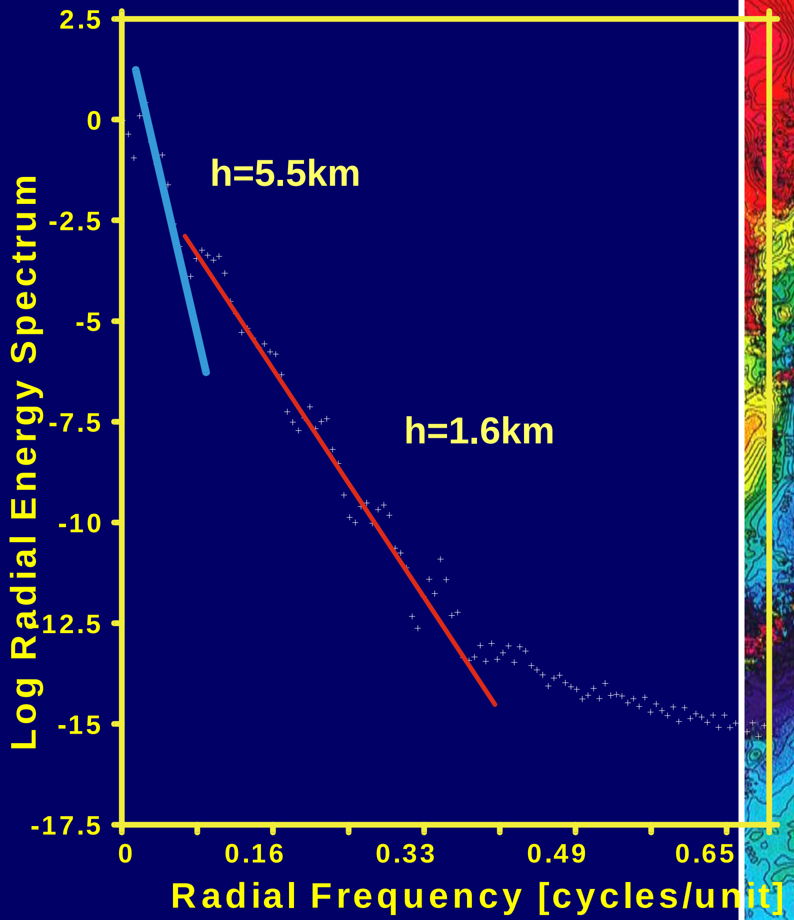


Multiple Prisms and Layers



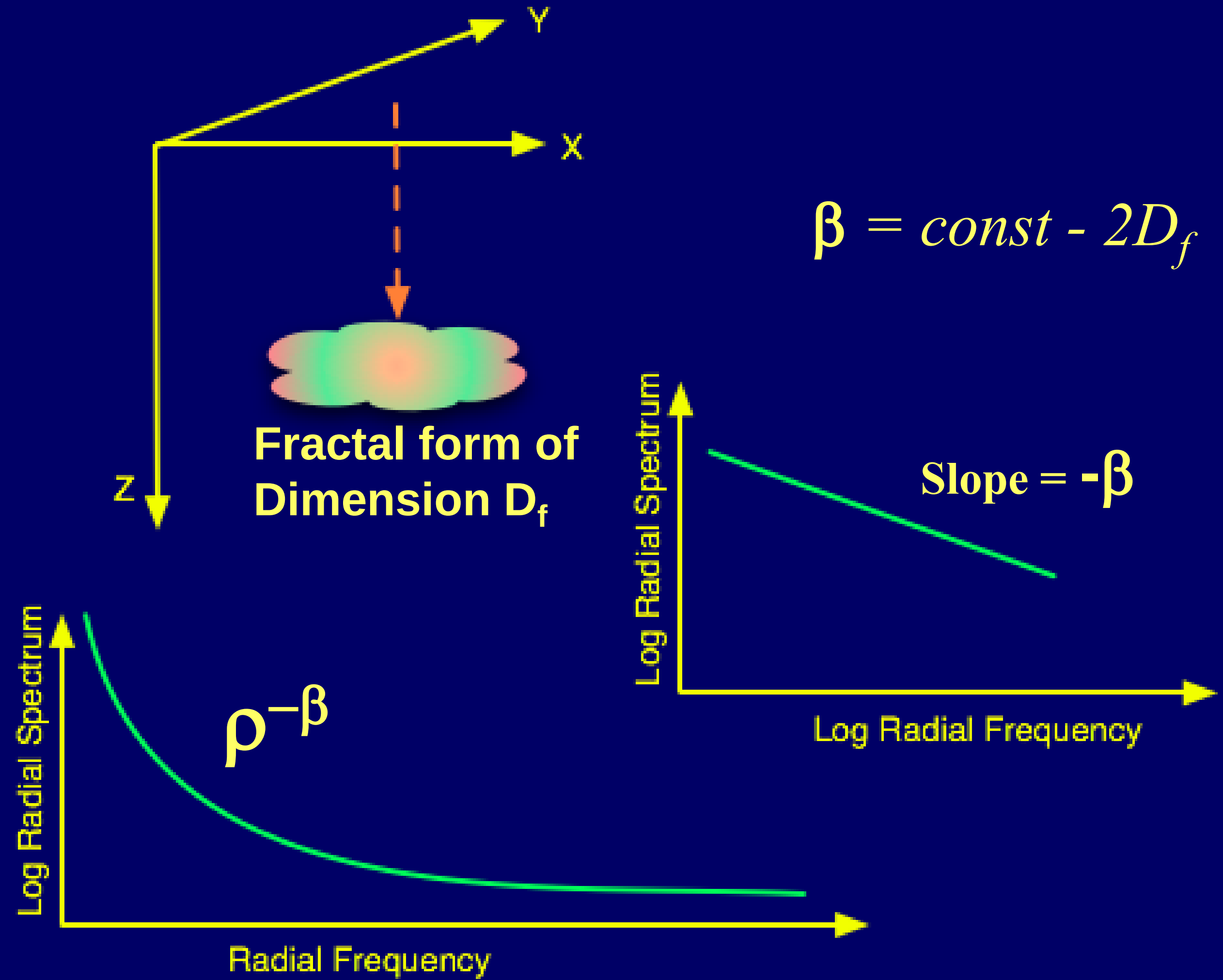
Multiple Prisms Model : Energy Spectrum





DETAIL OF LOW FREQUENCIES

General Fractal Model



Bad and Good Science

- Model A has behavior X
- System has behavior X
- System is of form Model A

Common and very bad science

- Have set of Models, A,B,C
  - Behaviors, X,Y,Z
- System has behavior X
- System may be of form A, is not of form B or C

Good Science

- Falsifiable Hypothesis
  - You might even be right, but have no reason to know you are right.

Source Ambiguity

- Quarta, Fedi, de Santis, 2000
  - β_f may depend upon ratio of horizontal extent of source and sampling interval - *not* fractal
  - Tests basic assumptions of fractal distribution
    - Synthetic models exhibited good match to fractal
    - Real data failed fractal test
  - Must constrain scale of fractal range
    - Extension over too large a range incorrect

Scale Ranges for ESA Fractals

- Lovejoy, Pecknold, Schertzer 2001
- Anisotropic model
- Slopes invariant on anisotropy
- Scales:
  - Core dominated
  - Curie Isotherm dominated
  - Small - Spector and Grant models

Sedimentary Layers

- **History of layer**
  - Existing topography
    - Probably anisotroic fractal
  - Sedimentary process
    - Some thickness of material
  - Possible new weathering of new surface
    - Maybe new fractal process
  - Horizon is difference of 2 (maybe fractal) topographies
  - Thin, maybe disconnected, lenticualr bodies.

Thin body model applies

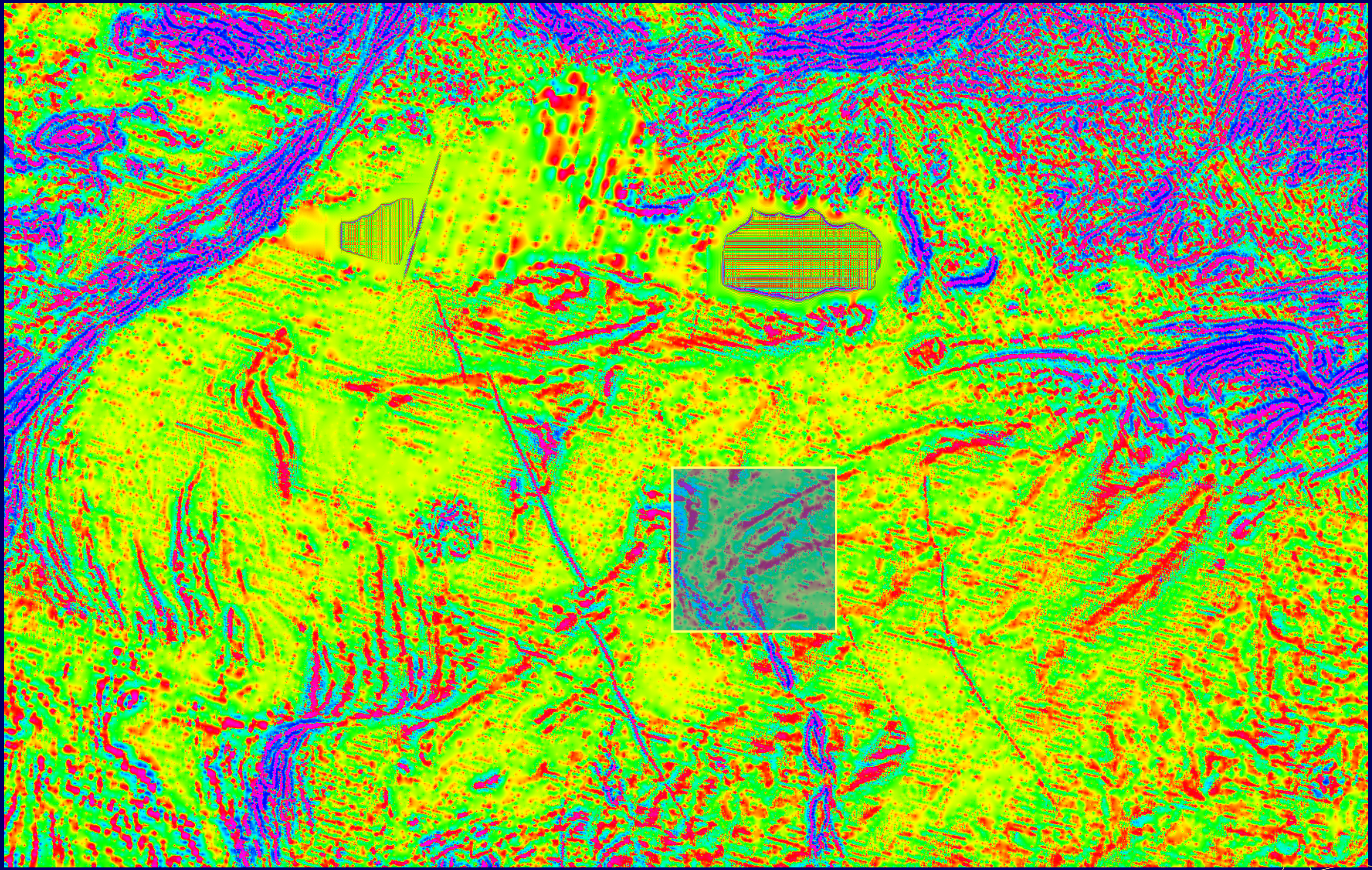
- Estimates depth well
- Spector and Grant + error within body

Pilkington, Gregotski, Todeoschuck

- Isotropic model of basement susceptibility distribution.
- Canadian Shield magnetic survey
  - Athabasca basin
  - Measured $\beta = 3$
  - Correct for f^{-3}
  - Downward continue until spectra flat
  - Yields correct depth estimate 1700m
- Lack of fractal correction (equivalent to Spector and Grant method)
  - Overestimates depth 2400m

Athabasca Basin

Data Quality

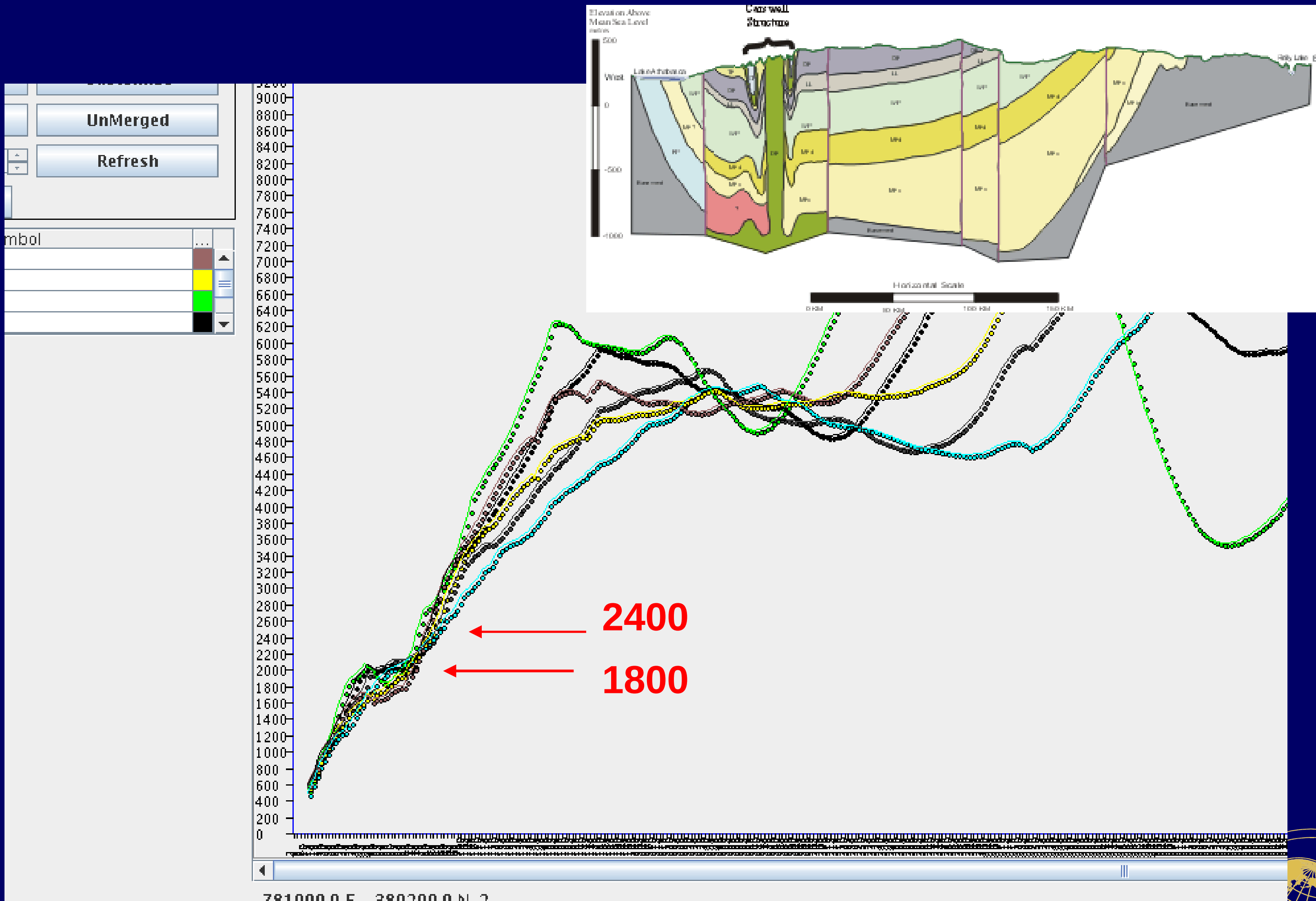


Second Vertical Derivative

Two sample areas

- Middle of basin, depth 1500m
- Exposed shield, depth = 0
- Flight height of 300m
- Exposed shield invalid test
  - Flight line spacing 812m
  - Over twice depth to source
  - However fractal analysis can extract depth
- Test - can MWT estimate correct depth?

Automatic MWT of Basin



Conclusions

- **Validates isotropic fractal model**
  - Small window size in geology smaller than dominated by Curie Isotherm models as Spector and Grant
  - So long as window size is small enough to avoid deep bodies
  - Too large window overestimates depth.
    - Contaminated by deeper bodies

Conculsions 2

- **MWT methodology avoids difficulties**
- **Estimates correct depth even in areas where previous use of poor window sizes failed**
- **Fractals remain an important model**
  - **Estimates of fractal dimensions should improve ESA methods**
  - **But must include anisotropy to work**
  - **Supporting evidence for estimating β needed**